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Prediction of Materials Charging in Magnetospheric Plasmas

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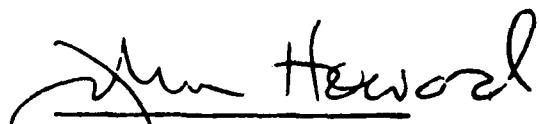


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FOR THE COMMANDER


John Howord
Chief Scientist

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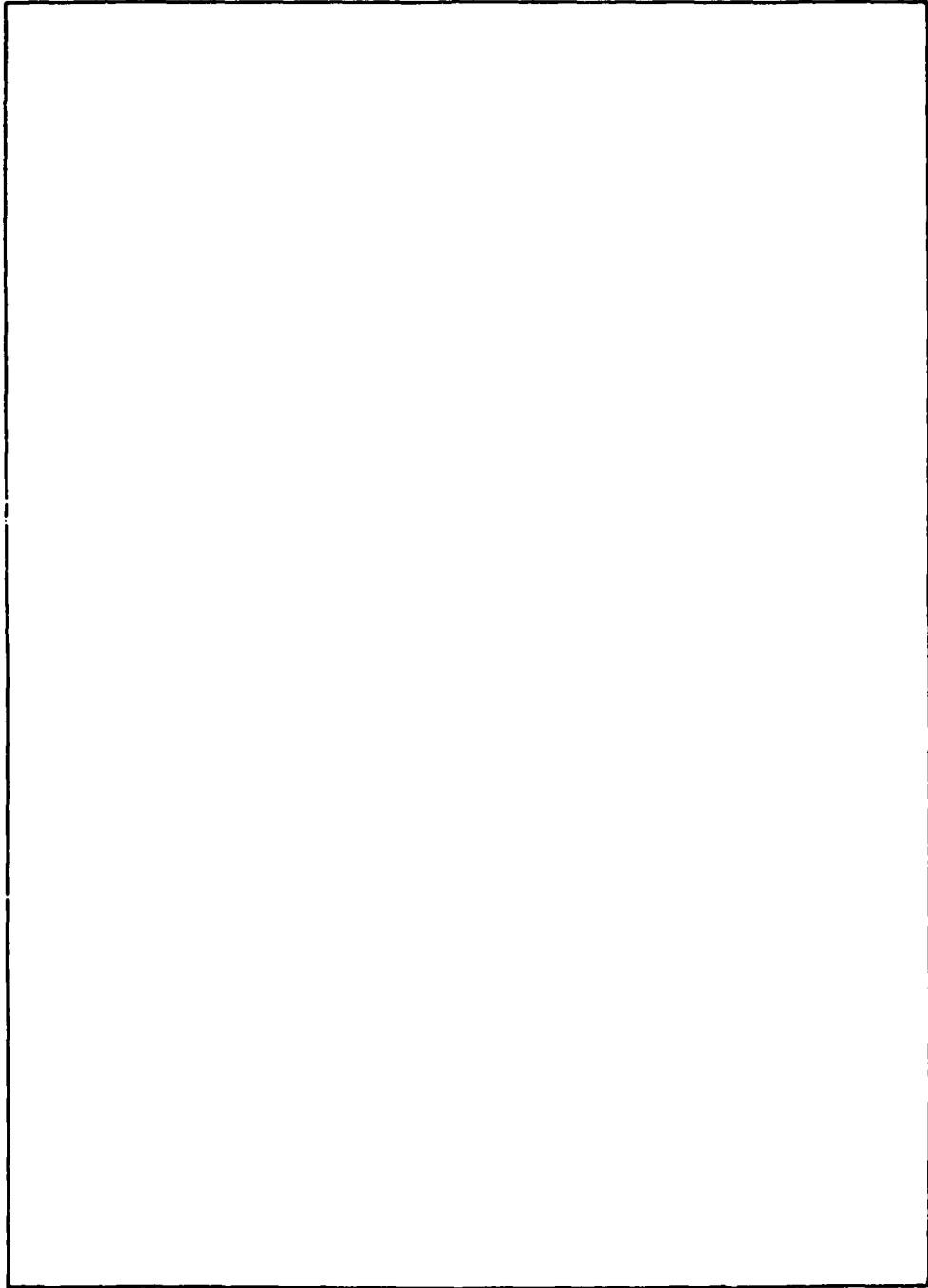
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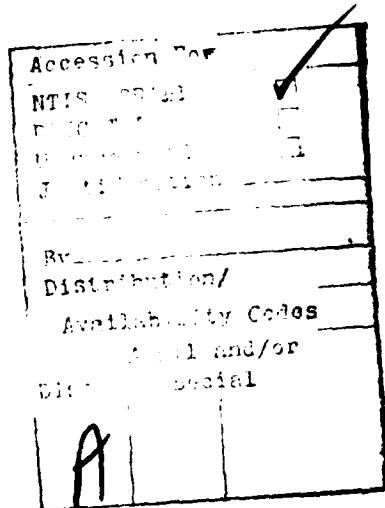
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Prediction of Materials Charging in Magnetospheric Plasmas

I. INTRODUCTION

It is often useful to obtain a quick approximation of the potential to which a material would charge in a geosynchronous orbit plasma. If one does not consider multidimensional effects and the effects of photoelectrons, a computation of the materials effects alone is possible.

This calculation is carried out by means of a computer code called MATCHG (Materials Charging). Given a plasma environment of specified electron and proton temperature and number density, the code calculates the potential to which a material will charge in this plasma. The principle of the method is to solve for the potential for which there is zero net current to the material surface. Included in the current balance are incident electron and proton (ion) currents, secondary electrons from both incident electrons and ions, backscattering, and current leakage to a substrate through bulk conductivity. In the formulation which includes bulk conductivity, the thickness of the material must be specified. The secondary-emission yield curves, as a function of energy, are calculated in the code.

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2. MATERIALS

The materials for which calculations were carried out are those typically used on satellite surfaces (see Table 1). "SOLAR" refers to solar cell cover glass material; "WHITEN" to a nonconducting white paint; "YELLOWC" to a yellow conducting paint; "BLACKC" to a black conducting paint.

The MATCHG program charges a surface by means of the same material parameters that are used in NASCAP.¹ It finds one-dimensional thickness estimates of the equilibrium potentials for a surface immersed in a specified plasma environment. The full 3-D complexities of more realistic models are neglected.

In this report we present the main results from MATCHG. Graphical and linearized techniques are developed for approximating the results of the MATCHG equations. Runs were made with the plasma temperature T_e ($T_i = 2T_e$) as the independent variable. The densities were fixed at $n_e = n_i = 10^6 \text{ M}^{-3}$. The potentials were computed in three ranges:

1. Low temperatures (see Figure 1)
2. Intermediate temperatures (see Figure 2)
3. High temperatures (see Figure 3).

Runs were made to calculate the potential for a two-Maxwellian plasma case and the corresponding single Maxwellian plasma case. The parameters specify three types of plasmas: hot, cold, and average. These are shown in Table 2. The MATCHG results are given in Table 3 for Teflon and aluminum.

The intermediate temperature cases were done for all of the materials currently available in MATCHG (see Figure 2). Note that materials BLACKC and YELLOWC are identical, that is, they are represented in the program by the same material parameters. Also the materials WHITEN and Kapton are sufficiently near to YELLOWC and BLACKC that the final potentials are the same (to three decimal places).

3. CHARGING AT LOW TEMPERATURES

Spacecraft charging is most significant for plasma temperatures greater than 1 kv. It is of interest to study charging at lower plasma temperatures as well.

For nonsunlit conditions, Figure 1 shows the equilibrium potentials for a number of materials, both metals and dielectrics.

1. Katz, I. et al (1977) A Three Dimensional Dynamic Study of Electrical Charging in Materials, NASA CR-135256.

Table 1. Material Properties

Property	Gold	Stainless	Wires	Neutron	Uranium	Platinum	Graphite	Specimen	Specimen	Specimen	Specimen	Specimen	Specimen
1. Depth, cm	1.0	4.0	—	1.0	—	—	—	1.0	—	—	—	—	—
2. Rh. losses (GeV)	1.0	1.7	—	1.1	—	—	—	1.0	—	—	—	—	—
3. Conductivity (mho/cm)	—	1.1 ± 0.7	2.9 ± 0.4	—	—	—	—	—	—	—	—	—	—
4. Effective Atomic No.	70.0	10.0	3.0	—	—	—	—	70.0	—	—	—	—	—
5. Yield for Electrons (GeV _{MAX})	0.88	4.1	2.1	0.6	1.0	1.0	1.0	—	—	—	—	—	—
6. E _{MAX} (GeV)	0.5	0.41	0.1	1.0	0.2	0.2	0.1	—	—	—	—	—	—
7. Range ₁ (cm)	63.0	—	—	10.0	—	—	—	300.0	—	—	—	—	—
8. Exponent ₁	1.80	—	—	1.0	—	—	—	—	—	—	—	—	—
9. Range ₂ (cm)	64.0	2.6 ^a	2.0 ^a	0.6	1.0	0.4	0.4	100.0	7.48 ^b	100.0 ^b	90.0 ^b	94.0 ^b	2.6 ^b
10. Exponent ₂	0.7	20.8	—	1.0	—	—	—	—	—	—	—	—	—
11. Yield for 1 GeV Protons	0.11 ^c	6.244	0.4 ^c	—	0.4	0.41 ^c	0.4	0.3 ^c	0.33 ^c	0.33 ^c	0.344 ^c	0.34 ^c	0.4 ^c
12. Energy for MAX Yield (GeV)	137.0	260.0	140.0	—	140.0	140.0	140.0	140.0	140.0	140.0	140.0	140.0	140.0
13. Photon loss (μA cm ⁻²)	29.6	20.0	29.0	0.0	—	—	—	—	—	—	—	—	—
14. Surface Resistivity (ohms)	0.0	1.4 ± 0.7	0.0	0.0	0.0	0.0	0.0	—	—	—	—	—	—

^aIons tv (GeV cm⁻³)^bBased on $\rho = 1.7 \times 10^{-14} \text{ cm}^2 \text{ GeV}^{-1}$ ^cBased on $\rho = 1.7 \times 10^{-14} \text{ cm}^2 \text{ GeV}^{-1}$

Table 2. Double and Single Maxwellian Plasma Cases

1) <u>Hot Plasma</u>	
$n_{e1} = 0.06$	$n_{i1} = 0.06$
$n_{e2} = 0.14$	$n_{i2} = 0.14$
$T_{e1} = 1200 \text{ eV}$	$T_{i1} = 2000 \text{ eV}$
$T_{e2} = 10000 \text{ eV}$	$T_{i2} = 16000 \text{ eV}$
1-Maxwellian version:	$n_e = n_i = 0.20$
$T_e = 8000$	$T_i = 13000$
2) <u>Cold Plasma</u>	
$n_{e1} = 0.85$	$n_{i1} = 0.25$
$n_{e2} = 0.25$	$n_{i2} = 0.85$
$T_{e1} = 1000 \text{ eV}$	$T_{i1} = 300 \text{ eV}$
$T_{e2} = 4000 \text{ eV}$	$T_{i2} = 9000 \text{ eV}$
1-Maxwellian versions:	$n_e = n_i = 1.1$
$T_e = 1800$	$T_i = 8000$
3) <u>Average Plasma</u>	
$n_{e1} = 0.9$	$n_{i1} = 0.9$
$n_{e2} = 0.6$	$n_{i2} = 0.6$
$T_{e1} = 1500 \text{ eV}$	$T_{i1} = 427 \text{ eV}$
$T_{e2} = 4400 \text{ eV}$	$T_{i2} = 10000 \text{ eV}$
1-Maxwellian versions:	$n_e = n_i = 1.5$
$T_e = 2800$	$T_i = 7000$

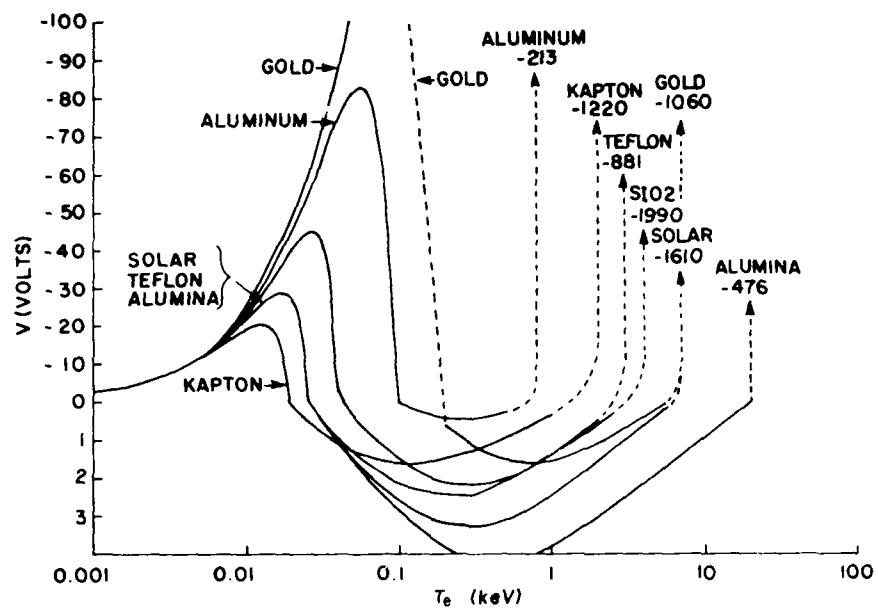


Figure 1. MATCHG Potentials for Low Temperatures

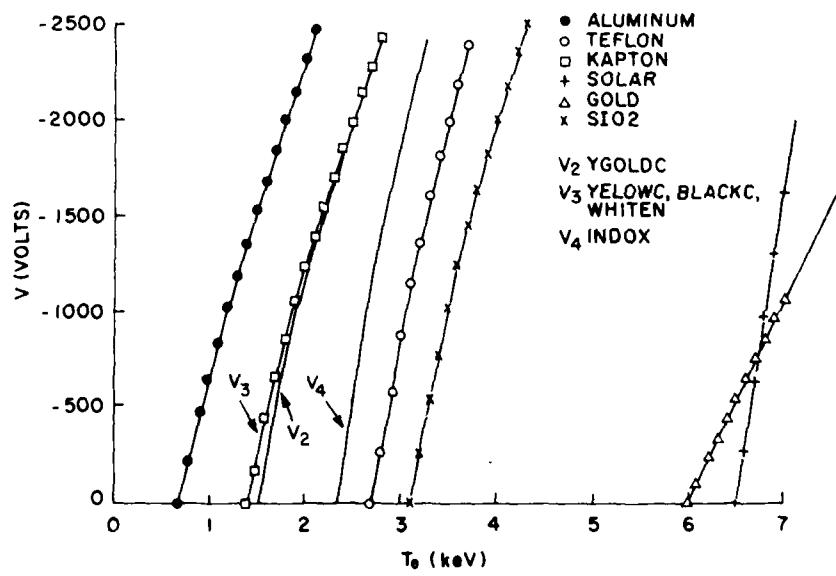


Figure 2. MATCHG Potentials for Intermediate Temperatures

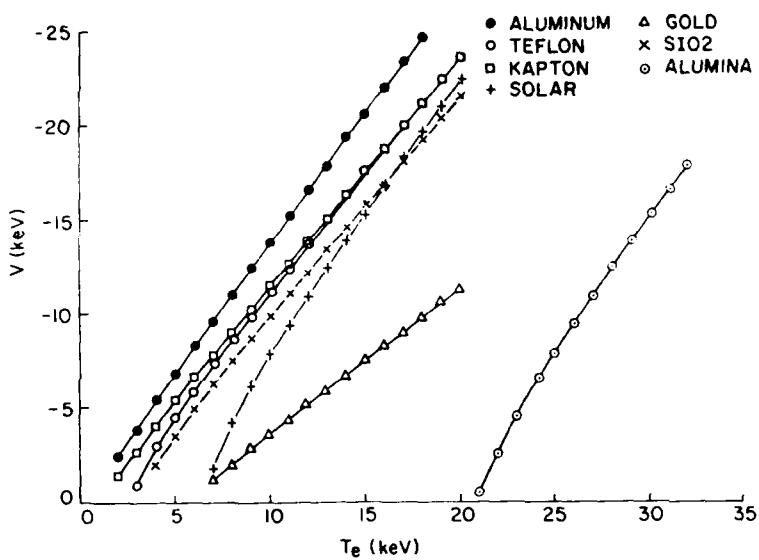


Figure 3. MATCHG Potentials for High Temperatures

At extremely low temperatures, up to 5 eV, all materials charge negatively to approximately the same potential. Between 10 and 100 eV, secondary emission properties become important and the potentials of different materials peak between -20 V for Teflon to -115 V for gold. Between about 20 eV and 200 eV plasma temperature, different materials charge to positive voltages. The positive voltages are all less than 5 eV, most around 2 eV.

In the temperature region between 10 and several hundred eV, materials potentials are either a few volts positive or tens of volts negative.

These results are significant in the design of electron spectrometers where the potentials produced by low-energy electrons impinging on dielectric materials is important.

4. EFFECT OF BULK CONDUCTIVITY

MATCHG includes the effect of the bulk conductivity current. In order to calculate this current, the potential difference across the charging surface must be known, that is, the voltage underlying the surface is needed.

Table 3. Single vs Double Maxwellian Results

Plasma Type	Material	Final Potential (keV)	Ratios (MAX / DBLMAX)	Initial Electron Current ($10^{-14} \text{ amp ps cm}^{-2}$)	Ratios (MAX / DBLMAX)
MAX - hot	Aluminum	-6.00		-0.479	
DBLMAX - hot	Aluminum	-5.94	1.01	-0.430	1.11
MAX - ave	Aluminum	-1.44		-2.12	
DBLMAX - ave	Aluminum	-1.44	0.88	-2.00	1.06
MAX - cold	Aluminum	-0.158		-1.25	
DBLMAX - cold	Aluminum	-0.0806	1.96	-1.14	1.10
MAX - hot	Teflon	-2.72		-0.479	
DBLMAX - hot	Teflon	-2.90	0.94	-0.430	1.11
MAX - ave	Teflon	-0.00071		-2.12	
DBLMAX - ave	Teflon	-0.00067	1.06	-2.00	1.06
MAX - cold	Teflon	+0.00127		-1.25	
DBLMAX - cold	Teflon	+0.00138	0.92	-1.14	1.10

To make a comparison to a NASCAP satellite model, the potential on the underlying conductor would be taken as the backing plate voltage. Table 4 shows the shade-side potentials for quasispheres in a fixed sun environment and the corresponding MATCHG voltages, including the bulk conductivity current. There is good agreement for all of the materials.

In Figure 4 we plot the MATCHG potentials vs the backing plate voltage for the case of Teflon in the nominal $T = 10, 20$ keV plasma. This figure shows that the minimum voltage $| -v |$ occurs when the backing voltage is 0.

In Figure 5 we show the lower limit MATCHG potentials (backing plate voltage set to 0) for a number of different materials. The plasma temperature, T_e ($T_i = 2T_e$), is the independent variable and the densities are fixed ($n_e = n_i = 10^6 \text{ M}^{-3}$). It is evident that those materials with large bulk conductivities (WHITEN, SI02) are depressed relative to the others, and SOLAR, with a small conductivity, charges up higher, that is, the larger the conductivity, the easier it is for charges to leak through the material and reduce the voltages.

Table 4. MATCHG Potentials vs NASCAP Quasisphere

Material	NASCAP Shade Potential	Quasisphere Conductor Potential	MATCHG	
			Backing Plate	Final Potential
Teflon	-8894	-7125	-7100	-8800
Kapton	-9210	-7378	-7380	-9280
SOLAR	-7820	-6264	-6260	-7820
SI02	-358	-279	-280	-360
WHITEN	-2997	-2395	-2390	-2990

Plasma: $T_e = 10$, $T_i = 20$ keV, $n_e = n_i = 10^6 \text{ M}^{-3}$
Sun is in direction (1, 1, 1)

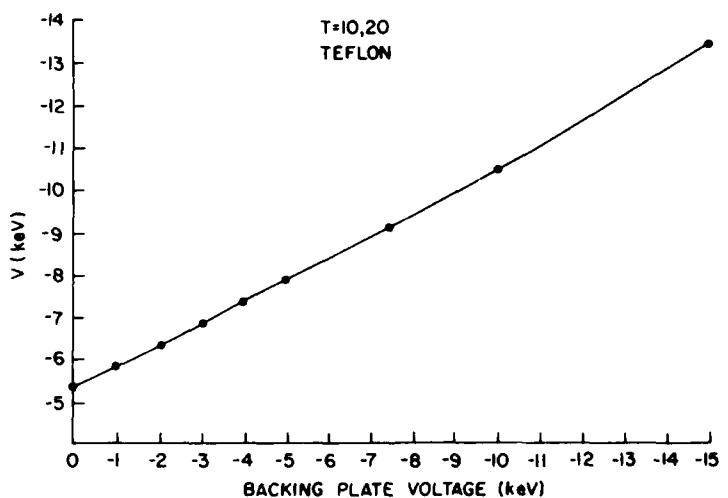


Figure 4. MATCHG Potentials vs Backing Voltage (Bulk Conductivity Included)

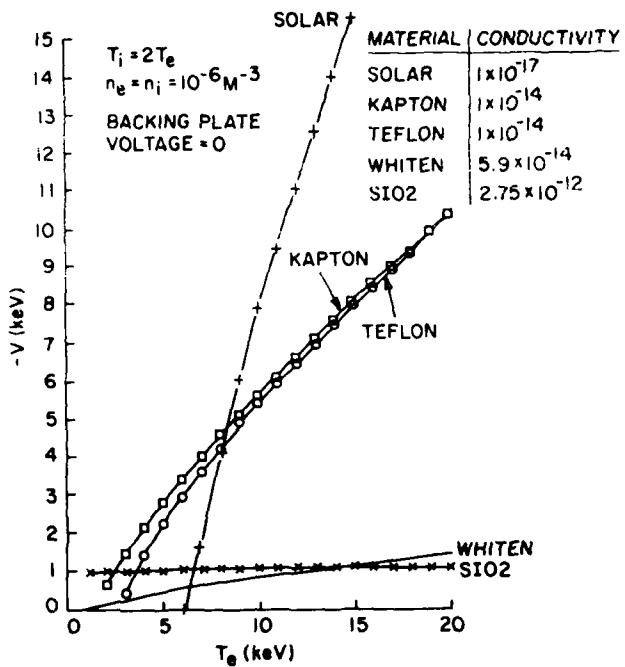


Figure 5. MATCHG Potentials (Bulk Conductivity Included)

Lower limit curves of this type would be directly applicable to cases where the spacecraft conductor does not charge and the surface lies in the shade (and the surface conductivity is negligible). For example, we can compare the numbers obtained for the SCATHA model for experiment SC1-3 (which lies in the shade) to the MATCHG results, as shown in Table 5. The agreement is very good.

When the bulk conductivity current is included in the computations, the steady-state potentials are determined by a balance between the current arriving from the plasma and that escaping through the surface to the backing plate. The final voltages reached will thus depend on the plasma density (which is related simply to the plasma current). This was not the case before when the bulk conductivity was neglected (see Figures 1 and 2). In Figure 6, we show the large changes that take place in the MATCHG potentials when the plasma density is varied by factors of 10 and 0.1 (for the case of a Teflon surface).

One can get a feeling for the effect of the ion temperature of the plasma on the MATCHG potentials from Figure 6. The plot shows the change in the voltage

Table 5. MATCHG Potentials vs SC1-3

Surface Material	4 Grid SCATHA Conductor Voltage -+6	Backing Plate Voltage	MATCHG Potential
Teflon	-452	0 +6	-457 -452
Kapton	-604	0 +6	-609 -604

Plasma Parameters ($T_e = 5$, $T_i = 6$ keV, $n_e = n_i = 10^{-6} M^{-3}$)

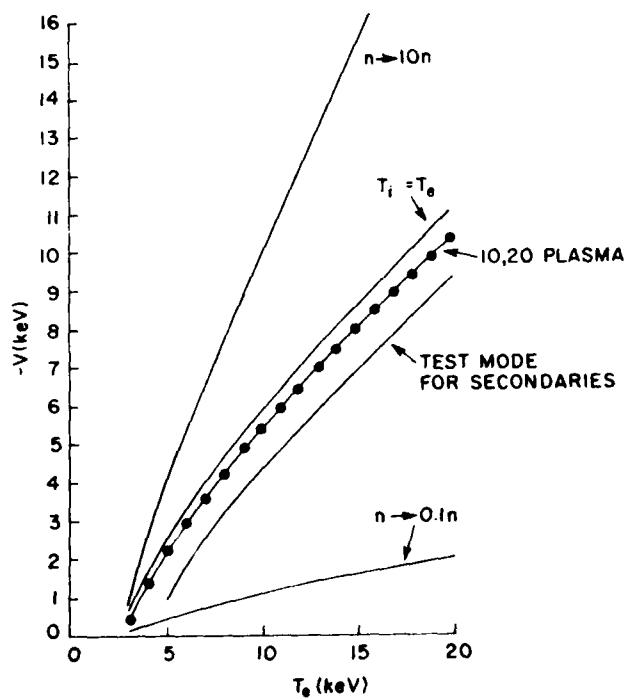


Figure 6. MATCHG Potentials-Teflon (Bulk Conductivity Included)

curve that occurs when we use the relation $T_e = T_i$ instead of the nominal $T_e = 2T_i$. There is only a small increase (~ 10 percent) in the equilibrium potentials.

An alternate formulation for the secondary emission was added, as a test mode, to MATCHG. In this new version, the Sternglass yield equation is used at low energies²

$$\delta/\delta_m = x e^{2(1-x)} \quad (x = E/E_m)$$

and the Wall formula at higher energies³

$$\delta = k x^{-0.725}$$

By imposing continuity of slope and value for the above two equations we set the transition energy $x = 3$ and the ratio $K/\delta_m = 1.54$. The MATCHG potentials, from runs using this test mode, are shown in Figure 6. The test equations result in a slight increase in the number of secondaries and thus the equilibrium potentials are decreased, as indicated in the figure.

5. GRAPHICAL AND LINEARIZED REPRESENTATION OF CHARGING EQUATIONS

We solve the circuit equations representing the charging calculation under the assumption that the source of current is a linear function of the source voltages. This procedure gives a fairly good approximation to the MATCHG results—the general features of the charging process are obtained (including final equilibrium voltage and time constant, which depends on conductivity, thickness of material, etc.).

6. CIRCUIT EQUATIONS

The charging calculations that are done in the MATCHG program can be represented by the circuit shown in Figure 7. The equation describing the voltage on the capacitor is, from the diagram,

2. Dekker, A.J. (1958) Solid State Physics, Academic Press, New York 6:279.
3. Wall, J.A. et al (1977) Results of Literature Search on Dielectric Properties and Electron Interaction Phenomena Related to Spacecraft Charging, Proceedings of the Spacecraft Charging Technology Conference, AFGL-TR-77-0051, AD A045 459.

$$\frac{dV}{dt} = \frac{i - i_R}{C} = \frac{i - (V - V_B)\sigma A/d}{K\epsilon_0 A/d} = \frac{1}{K\epsilon_0} (dj - (V - V_B)\sigma) \quad , \quad (1)$$

where $j = i/A$ is the new current density from the source (it does not include the bulk conductivity current). MATCHG calculates j , which is a complicated function of the plasma environment and the properties of the surface material. In Figure 8, we show the $j \sim j(V)$ dependence for Teflon in a plasma specified by $T_e = 5$ keV, $T_i = 6$, $n_e = 0.1 \text{ cm}^{-3}$, $n_i = 0.15$. It is evident that the j vs V curve is almost linear. Thus it will be a fairly good approximation to take

$$j = j_0 + J'V \quad , \quad (2)$$

where J' is some average slope. For simplicity, we could let

$$J' = - \frac{j_0}{V(\sigma = 0)} \quad , \quad (3)$$

where j_0 = initial net current density, $V(\sigma = 0)$ = equilibrium potential when $\sigma = 0$ which gives the straight line indicated in Figure 8 ($V(\sigma = 0) = 0$ and $j_0 < 0$ so that $J' < 0$). Substituting Eq. (2) into Eq. (1) gives

$$\frac{dV}{dt} = \frac{1}{K\epsilon_0} (dj_0 + V_B) + V \left(\frac{d}{K\epsilon_0} (J' - \sigma/d) \right) = A + B V \quad , \quad (4)$$

where we define

$$A = \frac{1}{K\epsilon_0} (dj_0 + V_B \sigma) \quad (5)$$

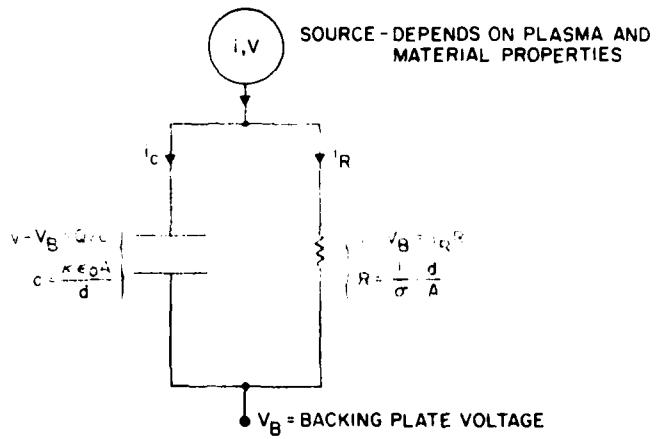
$$B = \frac{d}{K\epsilon_0} (J' - \sigma/d) < 0 \quad . \quad (6)$$

Equation (4) is easily solved. We write it as

$$\frac{dV}{(A + B V)} = dt \quad . \quad (7)$$

Integrating both sides of Eq. (7) gives

$$\frac{1}{B} \ln (A + B V) = t + C \quad . \quad (8)$$



κ = DIELECTRIC CONSTANT
 $\epsilon_0 = 8.85 \times 10^{-12}$
 d = THICKNESS OF MATERIAL
 A = AREA OF SURFACE
 σ = BULK CONDUCTIVITY

$\frac{dV}{dt} = \frac{i_C}{C}$
 $i_C = i - i_R$

EQUATIONS FOR CAPACITOR CHARGING

Figure 7. MATCHG Circuit Diagram

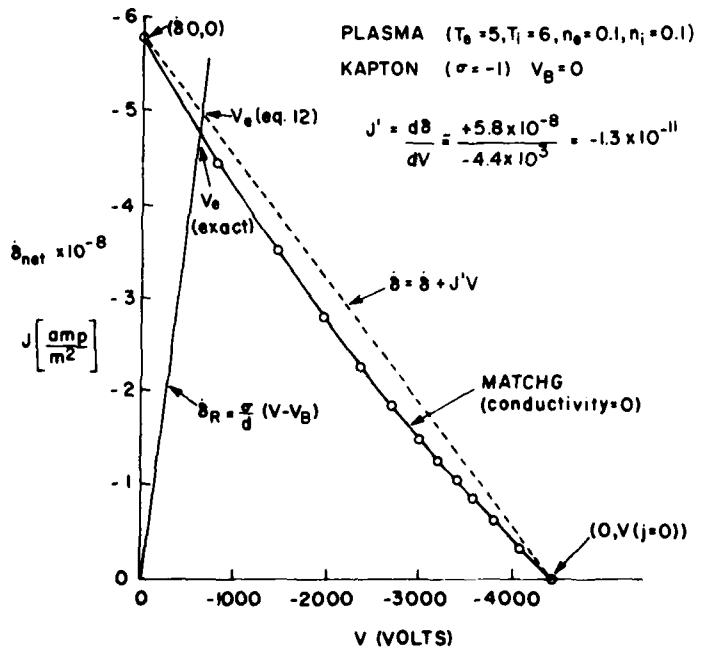


Figure 8. Net Current J vs V From MATCHG

The integration constant C can be evaluated by letting $t = 0$ in Eq. (8), that is,

$$C = \frac{1}{B} \ln (A + B V_o) , \quad (9)$$

where $V_o = V(t = 0)$. Putting Eq. (9) into Eq. (8) and solving for V leads to

$$V = \frac{A}{B} \left[\left(1 + \frac{B}{A} V_o \right) e^{Bt} - 1 \right] . \quad (10)$$

In Figure 9 we plot V from Eq. (10) (for the case $V_o = 0$) vs the time and the corresponding results obtained from MATCHG. The predicted curve is slightly higher. The time constant for charging up can be taken as

$$\tau = \frac{1}{B} = \frac{K\epsilon_o}{d} \frac{1}{(J' - \sigma/d)} = \frac{K\epsilon_o}{\sigma} \frac{1}{(J'd/\sigma - 1)} . \quad (11)$$

Check: when $J' = 0$ (constant source current), we have

$$\tau = \frac{K\epsilon_o}{\sigma} = \left(\frac{1}{\sigma} \frac{d}{A} \right) \left(\frac{K\epsilon_o A}{d} \right) = R C ,$$

which is what we would expect from elementary circuit theory. The final equilibrium potentials can be obtained by setting $\frac{dV}{dt} = 0$ in Eq. (4), or from Eq. (10) by setting $t = \infty$, since the exponential term goes to 0 ($B < 0$).

$$V_e = V(\infty) = - \frac{A}{B} = \frac{-(j_o + V_B \sigma/d)}{(J' - \sigma/d)} \quad (12)$$

or using Eq. (3),

$$= V(\sigma = 0) + \frac{\left(1 + \frac{V_B \sigma}{d} \right)}{\left(1 + \frac{V(\sigma = 0)}{j_o d} \cdot \sigma \right)} .$$

Check: when $\sigma = 0$ we recover the result that $V_e = V(\sigma = 0)$.

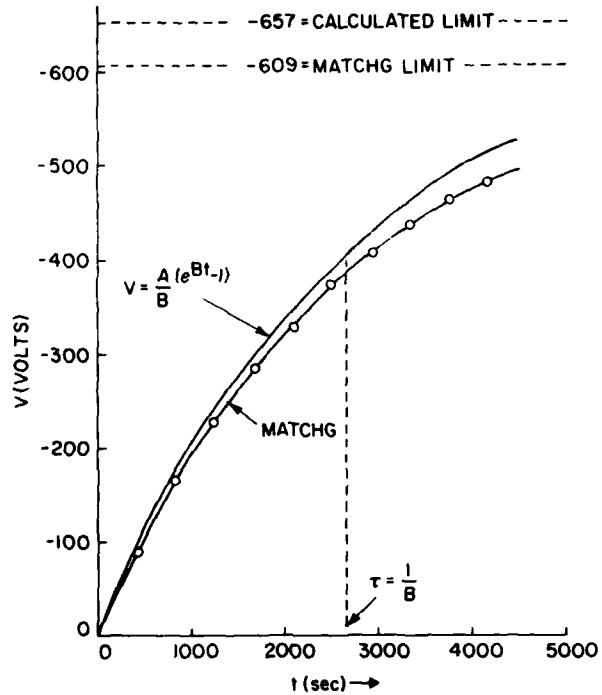


Figure 9. Calculated V vs Time From MATCHG

7. GRAPHICAL SOLUTION

As time increases, the source current, i , decreases and the conductivity current, i_R , increases in proportion to the voltage buildup on the capacitor. At equilibrium, the current to the capacitor, i_C , has decreased to 0 and thus $i = i_R$, or in terms of current densities we have

$$j = j_0 + J'V_e = j_R = \frac{\sigma}{d}(V_e - V_B) .$$

Solving the condition for V_e again yields the Eq. (12). One could solve the above equations graphically by finding the intersection of the straight lines representing j and j_R on the (j, V) plot (see Figure 8). In fact, an exact solution would be determined by the intersection of the j_R line and the MATCHG $j(V)$ curve. The effect of changing σ , d , and V_B is clear from this construction. One just changes the slope and intercept of the j_R line and finds the new intersection point giving V_e .

8. NUMBERS FOR T = 5.6 CASE

Kapton

$$\sigma = 10^{-14} , \quad K = 3.5 , \quad d = 1.27 \times 10^{-4} ,$$

$$V(\sigma = 0) = -4.43 \times 10^3 , \quad j_o = -5.83 \times 10^{-8} , \quad V_B = 0$$

$$J' = - \frac{(-5.83 \times 10^{-8})}{(-4.43 \times 10^{-3})} = -1.3 \times 10^{-11}$$

$$A = \frac{dj_o}{K\epsilon_o} = \frac{(1.27 \times 10^{-4})(-5.83 \times 10^{-8})}{(3.5)(8.85 \times 10^{-12})} = -0.239$$

$$B = \frac{d}{K\epsilon_o} (J' - \sigma/d)$$

$$= \frac{1.27 \times 10^{-4}}{(3.5)(8.85 \times 10^{-12})} \left(-1.3 \times 10^{-11} - \frac{10^{-14}}{1.27 \times 10^{-4}} \right)$$

$$= 3.64 \times 10^{-4}$$

$$V_e = \frac{-A}{B} = - \frac{(-0.239)}{(-3.64 \times 10^{-4})} = -657 .$$

$$\tau = \frac{K\epsilon_o}{\sigma} \frac{1}{\left(J' \frac{d}{\sigma} - 1 \right)}$$

$$= \frac{(3.5)(8.85 \times 10^{-12})}{10^{-14}} \frac{1}{\left(\frac{(-1.3 \times 10^{-11})(1.27 \times 10^{-4})}{10^{-14}} - 1 \right)}$$

$$= 2.66 \times 10^3 ,$$

from MATCHG run, $V_e = -609$.

